AP Test Review

Name: ______________________

Date: ______________ Period: ____

Question Type #5- Particle Motion

What you should be able to do:

- Approximate the derivative (slope, rate of change, average rate of change) using a difference quotient.
- Use Riemann sums (left, right, midpoint) or a trapezoidal approximation to approximate the value of a definite integral using values in the table (typically with uneven subintervals).
- Calculate average value
- Explain the meaning of a definite integral in the context of the problem
- Calculate a tangent line approximation (local linear approximation)
- Questions about Rolle's theorem (the MVT when $f'(c)=0$), MVT, IVT, etc. including theory questions

Do not: Use a calculator to find a regression equation and then use that to answer parts of the question. (While finding them ids perfectly good mathematics, regression equations are not one of the four things students may do with their calculator and give only an approximation of our function.)

Multiple Choice Practice: Non Calculator

x	2	5	7	8
$f(x)$	10	30	40	20

1. The function f is continuous on the closed interval [2, 8] and has values that are given in the table above. Using the subintervals [2, 5], [5, 7], and [7, 8], what is the trapezoidal approximation of $\int_2^8 f(x)dx$?

$\frac{1}{2}(3)(10+30)+\frac{1}{2}(2)(30+40)+\frac{1}{2}(1)(40+20)$

a) 110

b) 130

(c) 160 — circled

$60+70+30$

d) 190

160

e) 210

2. The table below gives values of f, f', g and g' at selected values of x. If $h(x)=f(g(x))$, then $h'(1)=$

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
-1	6	5	3	-2
1	3	-3	-1	2
3	1	-2	2	3

a) 5

b) 6

c) 9

(d) 10 — circled

e) 12

$h'(x)=f'(g(x))\cdot g'(x)$

$h'(1)=f'(g(1))\cdot g'(1)$

$=f'(-1)\cdot 2$

$=5\cdot 2$

$=10$

increasing concave down

3. For all x in the closed interval [2, 5], the function f has a positive first derivative and a negative second derivative. Which of the following could be a table of values for f?

a)

x	$f(x)$
2	7
3	9
4	12
5	16

+2, +3, +4

(b))

x	$f(x)$
2	7
3	11
4	14
5	16

+4, +3, +2

c)

x	$f(x)$
2	16
3	12
4	9
5	7

d)

x	$f(x)$
2	16
3	14
4	11
5	7

e)

x	$f(x)$
2	16
3	13
4	10
5	7

c, d, and e are all decreasing

<u>Multiple Choice Practice</u>: Calculator

x	0	0.5	1.0	1.5	2.0
$f(x)$	3	3	5	8	13

4. A table of values for a continuous function f is shown above. If <u>four equal subintervals</u> of [0, 2] are used, which of the following is the trapezoidal approximation of $\int_0^2 f(x)dx$?

$\frac{1}{2}(.5)[(13+3)+(3+5)+(5+8)+(8+13)]$

$\frac{1}{4}(48)$

a) 8
(b)) 12
c) 16
d) 24
e) 32

5. The function f is continuous on the closed interval [2, 14] and has values as shown in the table below. Using the sub intervals [2, 5], [5, 10], and [10, 14], what is the approximation of $\int_2^{14} f(x)dx$ found by using a right Riemann sum?

a) 296
b) 312
c) 343
(d)) 374
e) 390

$4(30)+5(34)+3(28)$

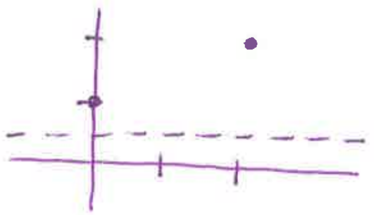
x	2	5	10	14
$f(x)$	12	28	34	30

x	0	1	2
$f(x)$	1	k	2

6. The function f is continuous on the closed interval $[0, 2]$ and has values that are given in the table above. The equation $f(x) = \frac{1}{2}$ must have at least two solutions in the interval $[0, 2]$ if $k =$

Hint: draw pictures!

a) 0
b) $\frac{1}{2}$
c) 1
d) 2
e) 3



7.

x	-4	-3	-2	-1	0	1	2	3	4
$g'(x)$	2	3	0	-3	-2	-1	0	3	2

The derivative g' of a function g is continuous and has exactly two zeros. Selected values of g' are given in the table above. If the domain of g is the set of all real numbers, then g is decreasing on which of the following intervals?

a) $-2 \le x \le 2$ only
b) $-1 \le x \le 1$ only
c) $x \ge -2$
d) $x \ge 2$ only
e) $x \le -2$ or $x \ge 2$

x	0	1	2	3	4
$f(x)$	2	3	4	3	2

8. The function f is continuous and differentiable on the closed interval $[0, 4]$. The table above gives selected values of f on this interval. Which of the following statements must be true?

a) The minimum value of f on $[0, 4]$ is 2.
b) The maximum value of f on $[0, 4]$ is 4.
c) $f(x) > 0$ for $0 < x < 4$
d) $f'(x) < 0$ for $2 < x < 4$
e) There exists c, with $0 < c < 4$, for which $f'(c) = 0$.

ANSWERS: 1. C 2. D 3. B 4. B 5. D 6. A 7. A 8. E

Work for 2007 Question #3

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Question 2

t (hours)	0	1	3	4	7	8	9
$L(t)$ (people)	120	156	176	126	150	80	0

Concert tickets went on sale at noon $(t = 0)$ and were sold out within 9 hours. The number of people waiting in line to purchase tickets at time t is modeled by a twice-differentiable function L for $0 \le t \le 9$. Values of $L(t)$ at various times t are shown in the table above.

(a) Use the data in the table to estimate the rate at which the number of people waiting in line was changing at 5:30 P.M. $(t = 5.5)$. Show the computations that lead to your answer. Indicate units of measure.

(b) Use a trapezoidal sum with three subintervals to estimate the average number of people waiting in line during the first 4 hours that tickets were on sale.

(c) For $0 \le t \le 9$, what is the fewest number of times at which $L'(t)$ must equal 0 ? Give a reason for your answer.

(d) The rate at which tickets were sold for $0 \le t \le 9$ is modeled by $r(t) = 550te^{-t/2}$ tickets per hour. Based on the model, how many tickets were sold by 3 P.M. $(t = 3)$, to the nearest whole number?

(a) $L'(5.5) \approx \dfrac{L(7) - L(4)}{7 - 4} = \dfrac{150 - 126}{3} = 8$ people per hour

$2 : \begin{cases} 1 : \text{estimate} \\ 1 : \text{units} \end{cases}$

(b) The average number of people waiting in line during the first 4 hours is approximately

$$\frac{1}{4}\left(\frac{L(0) + L(1)}{2}(1 - 0) + \frac{L(1) + L(3)}{2}(3 - 1) + \frac{L(3) + L(4)}{2}(4 - 3)\right)$$

$= 155.25$ people

$2 : \begin{cases} 1 : \text{trapezoidal sum} \\ 1 : \text{answer} \end{cases}$

(c) L is differentiable on $[0, 9]$ so the Mean Value Theorem implies $L'(t) > 0$ for some t in $(1, 3)$ and some t in $(4, 7)$. Similarly, $L'(t) < 0$ for some t in $(3, 4)$ and some t in $(7, 8)$. Then, since L' is continuous on $[0, 9]$, the Intermediate Value Theorem implies that $L'(t) = 0$ for at least three values of t in $[0, 9]$.

$3 : \begin{cases} 1 : \text{considers change in sign of } L' \\ 1 : \text{analysis} \\ 1 : \text{conclusion} \end{cases}$

OR

The continuity of L on $[1, 4]$ implies that L attains a maximum value there. Since $L(3) > L(1)$ and $L(3) > L(4)$, this maximum occurs on $(1, 4)$. Similarly, L attains a minimum on $(3, 7)$ and a maximum on $(4, 8)$. L is differentiable, so $L'(t) = 0$ at each relative extreme point on $(0, 9)$. Therefore $L'(t) = 0$ for at least three values of t in $[0, 9]$.

OR

$3 : \begin{cases} 1 : \text{considers relative extrema of } L \text{ on } (0, 9) \\ 1 : \text{analysis} \\ 1 : \text{conclusion} \end{cases}$

[*Note: There is a function L that satisfies the given conditions with $L'(t) = 0$ for exactly three values of t.*]

(d) $\int_0^3 r(t)\,dt = 972.784$

There were approximately 973 tickets sold by 3 P.M.

$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits and answer} \end{cases}$

Work for 2008 Question #2

L is twice differentiable, so both L and L' are continuous.

(a) $L'(5.5) \approx \dfrac{L(7)-L(4)}{7-4} = \dfrac{150-126}{3} = \boxed{8\,\dfrac{people}{hour}}$

(b) Average number of people in line on [0,4] is $\dfrac{1}{4-0}\int_0^4 L(t)\,dt$.

Using a trapezoidal approximation for the integral:

$$\frac{1}{4}\left[\frac{1}{2}(1)(156+120)+\frac{1}{2}(2)(176+156)+\frac{1}{2}(1)(126+176)\right] people \text{ or } \boxed{155.25\ people}$$

(c) By the Mean Value Theorem, there is an

$x_1 \in (0,1)$ where $f\,'(x_1) = \dfrac{156-120}{1-0} = 36$

$x_2 \in (1,3)$ where $f\,'(x_2) = \dfrac{176-156}{2} = 10$

$x_3 \in (3,4)$ where $f\,'(x_3) = \dfrac{126-176}{1} = -50$

$x_4 \in (4,7)$ where $f\,'(x_4) = \dfrac{150-126}{3} = 8$

$x_5 \in (7,8)$ where $f\,'(x_5) = \dfrac{80-150}{1} = -70$

$x_6 \in (8,9)$ where $f\,'(x_6) = \dfrac{0-80}{1} = -80$

Since $f\,'$ is continuous, when $f\,'$ changes signs on an interval, then $f\,'=0$ somewhere on that interval by the Intermediate Value Theorem. This happens on the intervals (x_2,x_3), (x_3,x_4), and (x_4,x_5). So, $f\,'=0$ at least $\boxed{3 \text{ times}}$.

(d) $r(t) = 550te^{-t/2}\ \dfrac{tickets}{hour}$

There where no tickets sold at time $t=0$, so the number of tickets sold by 3pm is

$\int_0^3 r(t)\,dt = 972.784$ or, to the nearest ticket, $\boxed{973 \text{ tickets}}$

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Question 5

t (minutes)	0	2	5	7	11	12
$r'(t)$ (feet per minute)	5.7	4.0	2.0	1.2	0.6	0.5

The volume of a spherical hot air balloon expands as the air inside the balloon is heated. The radius of the balloon, in feet, is modeled by a twice-differentiable function r of time t, where t is measured in minutes. For $0 < t < 12$, the graph of r is concave down. The table above gives selected values of the rate of change, $r'(t)$, of the radius of the balloon over the time interval $0 \le t \le 12$. The radius of the balloon is 30 feet when $t = 5$. (Note: The volume of a sphere of radius r is given by $V = \frac{4}{3}\pi r^3$.)

(a) Estimate the radius of the balloon when $t = 5.4$ using the tangent line approximation at $t = 5$. Is your estimate greater than or less than the true value? Give a reason for your answer.

(b) Find the rate of change of the volume of the balloon with respect to time when $t = 5$. Indicate units of measure.

(c) Use a right Riemann sum with the five subintervals indicated by the data in the table to approximate $\int_0^{12} r'(t)\,dt$. Using correct units, explain the meaning of $\int_0^{12} r'(t)\,dt$ in terms of the radius of the balloon.

(d) Is your approximation in part (c) greater than or less than $\int_0^{12} r'(t)\,dt$? Give a reason for your answer.

Solution	Points
(a) $r(5.4) \approx r(5) + r'(5)\Delta t = 30 + 2(0.4) = 30.8$ ft Since the graph of r is concave down on the interval $5 < t < 5.4$, this estimate is greater than $r(5.4)$.	2 : { 1 : estimate; 1 : conclusion with reason
(b) $\frac{dV}{dt} = 3\left(\frac{4}{3}\right)\pi r^2 \frac{dr}{dt}$ $\left.\frac{dV}{dt}\right\|_{t=5} = 4\pi(30)^2 2 = 7200\pi \text{ ft}^3/\text{min}$	3 : { 2 : $\frac{dV}{dt}$; 1 : answer
(c) $\int_0^{12} r'(t)\,dt \approx 2(4.0) + 3(2.0) + 2(1.2) + 4(0.6) + 1(0.5)$ $= 19.3$ ft $\int_0^{12} r'(t)\,dt$ is the change in the radius, in feet, from $t = 0$ to $t = 12$ minutes.	2 : { 1 : approximation; 1 : explanation
(d) Since r is concave down, r' is decreasing on $0 < t < 12$. Therefore, this approximation, 19.3 ft, is less than $\int_0^{12} r'(t)\,dt$.	1 : conclusion with reason
Units of ft^3/min in part (b) and ft in part (c)	1 : units in (b) and (c)

Work for 2007 Question #5

(a) The linearization/tangent line equation is: $y - 30 = 2(t - 5)$.

This means that r(5.4): $y - 30 = 2(5.4 - 5)$

$y - 30 = 2(.4)$

$r(5.4) = 30.8$ feet

This estimate is greater than the true value at $t = 5.4$ because r is concave down in the interval $5 < t < 5.4$

(b) Since $V=\frac{4}{3}\pi r^3$, then $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

When $t = 5$, $\frac{dV}{dt} = 4\pi(30)^2(2) = \boxed{7200\pi \ \frac{ft^3}{min}}$

(c) right Riemann sum: $\int_0^{12} r'(t)dt \approx \boxed{2(4)+3(2)+2(1.2)+4(0.6)+1(0.5)}$ or $8 + 6 + 2.4 + 2.4 + .5 = \boxed{19.3 \text{ feet}}$

This means that the radius has grown about 19.3 feet from $t = 0$ minutes until $t = 12$ minutes because this integral is the change in the radius, in feet, from $t = 0$ minutes to $t = 12$ minutes..

(d) Since r is concave down on the interval $0 < t < 12$, then r' is decreasing on the interval. The approximation is $\boxed{\textit{less than}}$ the true value of the integral because the right Riemann sum underestimates the true value when the curve is decreasing.