AP Test Review

Name: ____________________
Date: ______________ Period: _____

Question Type #1- Interpreting Graphs

"Here's the graph of the derivative, tell me things about the function."

What you should be able to do:

- Read information about the function from the graph of the derivative. This may be approached as a derivative techniques or antiderivative techniques.
- Find where the function is increasing or decreasing.
- Find and justify extreme values (1st and 2nd derivative tests, Closed interval test, aka. Candidates' test).
- Find and justify points of inflection.
- Find slopes (second derivatives, acceleration) from the graph.
- Write an equation of a tangent line.
- Evaluate Riemann sums from geometry of the graph only.
- FTC: Evaluate integral from the area of regions on the graph.
  FTC: The function, $g(x)$, maybe defined by an integral where the given graph is the graph of the integrand, $f(t)$, so you should know that if $g(x) = g(a) + \int_a^t f(t)dt$ then $g'(x) = f(x)$ and $g''(x) = f'(x)$.

The ideas and concepts that can be tested with this type question are numerous. The type appears on the multiple-choice exams as well as the free-response. They have accounted for almost 25% of the points available on recent test.

Multiple Choice Practice: Non Calculator

1. The graph of f', the derivative of the function f, is shown to the right. Which of the following statements is true about f?

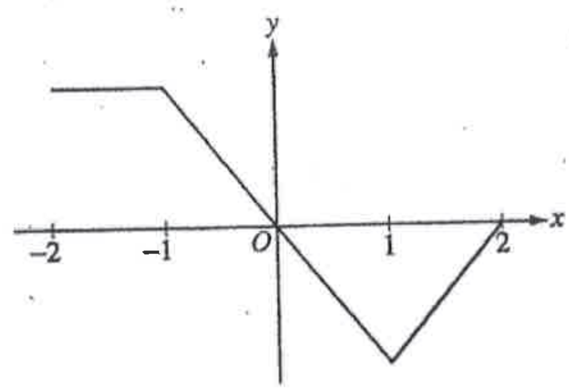
   a) f is not differentiable at $x = -1$ and $x = 1$

   b) f is decreasing for $-1 \leq x \leq 1$

   c) f is increasing for $1 \leq x \leq 2$

   d) f has a local maximum at $x = 0$

   e) f has a local minimum at $x = 0$



Graph of f'

2. The function f is shown below. At which value of x is f continuous, but not differentiable?

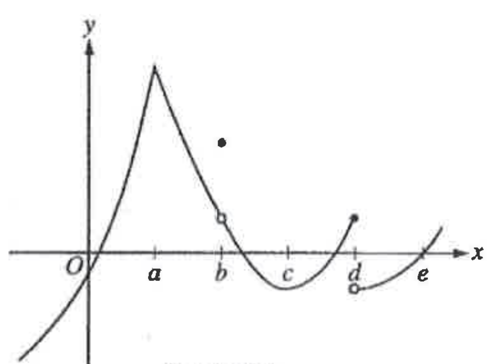
   a) a

   b) b

   c) c

   d) d

   e) e



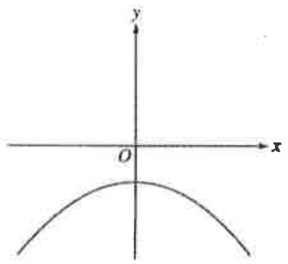
Graph of f

negative concave down

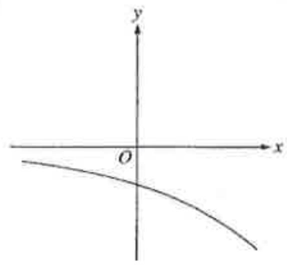
3. The function f has the property that $f(x) < 0, f'(x) > 0, f''(x) < 0$ for all real values x. Which of the following could be the graph of f?

increasing

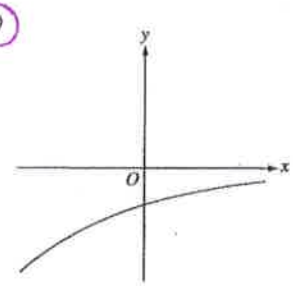
(A)



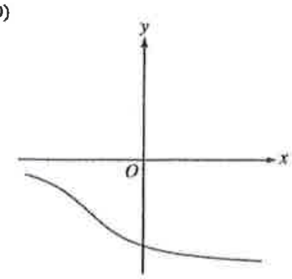
(B)



(C)



(D)



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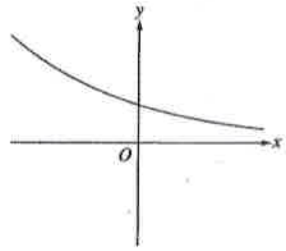


4. The second derivative of the function f is given by $f''(x) = x(x-a)(x-b)^2$. The graph of f'' is shown below. For what values of x does the graph of f have a point of inflection?

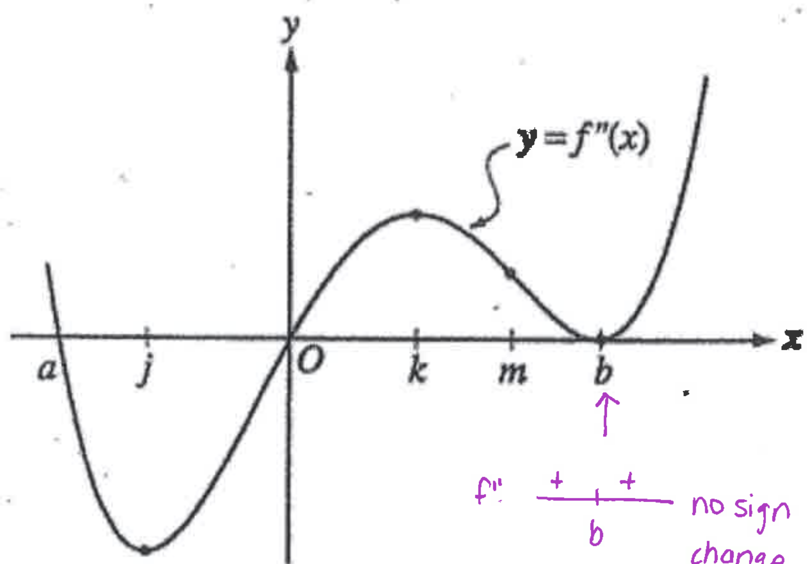
a) 0 and a only

b) 0 and m only

c) b and j only

d) 0, a, and b

e) b, j, and k



f'' + + b no sign change

5. The graph of f', the derivative of f, is the line shown in the figure to the right. If $f(0) = 5$, then $f(1) =$

a) 0

b) 3

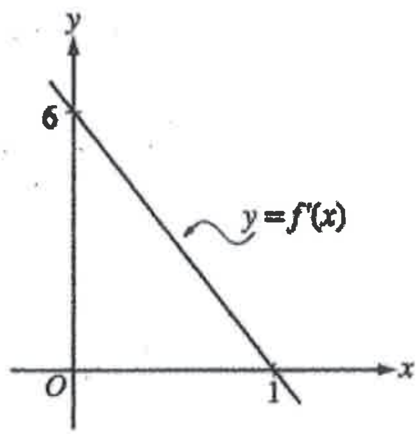
c) 6

d) 8

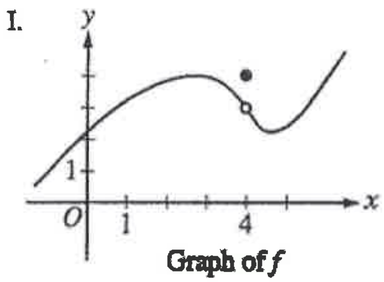
e) 11

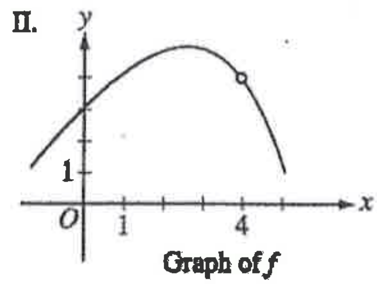
$$\int_0^1 f'(x)\,dx = f(1) - f(0)$$

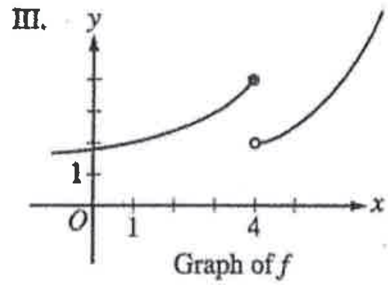
$$3 = f(1) - 5$$



6. For which of the following does $\lim_{x \to 4} f(x)$ exist?







a) I only

b) II only

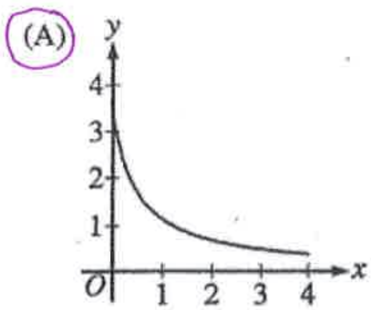
c) III only

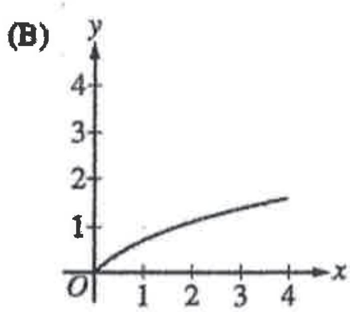
d) I and II only

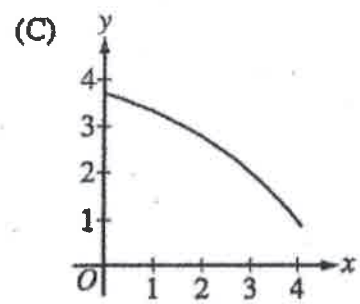
e) I and III only

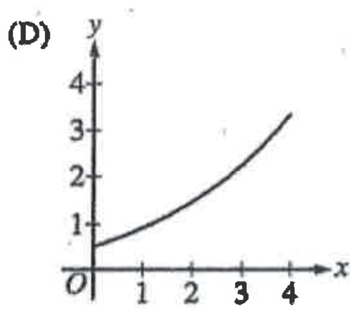
$\lim_{x \to 4} f(x)$ exists if $\lim_{x \to 4^-} f(x) = \lim_{x \to 4^+} f(x)$

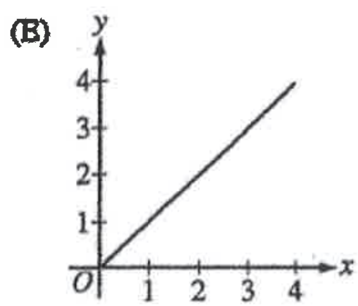
7. If a trapezoidal sum overapproximates $\int_0^4 f(x)\,dx$, and a right Riemann sum underapproximates $\int_0^4 f(x)\,dx$, which of the following could be the graph of $y = f(x)$?







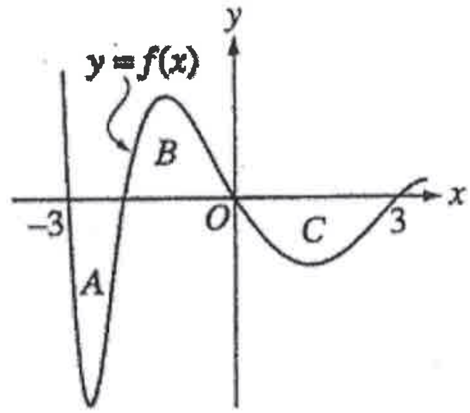




Trap. over approximates if f(x) is concave up

Right under approximates if f(x) is decreasing

8. The regions A, B, and C in the figure below are bounded by the graph of the function f and the x-axis. If the area of each region is 2, what is the value of $\int_{-3}^{3}(f(x)+1)dx$?



a) -2

b) -1

c) 4

d) 7

e) 12

$\int_{-3}^{3} f(x)\,dx + \int_{-3}^{3} 1\,dx$

$(-2+2-2) + x\Big|_{-3}^{3}\,dx$

$-2 + (3-(-3))$

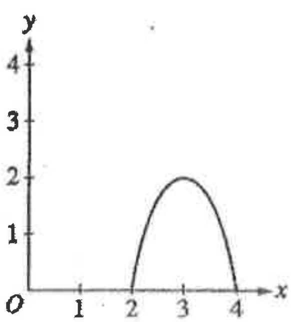
$-2 + (3+3)$

$-2+6$

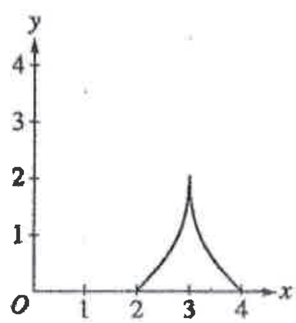
9. On the closed interval $[2, 4]$, which of the following could be the graph of a function f with the property that $\underbrace{\frac{1}{4-2}\int_2^4 f(t)dt}_{} = 1$?

represents the average value of f(x) on the interval [2, 4]

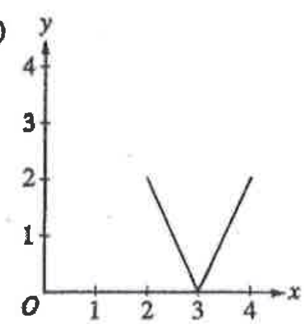
(A)



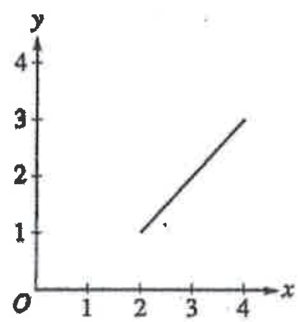
(B)



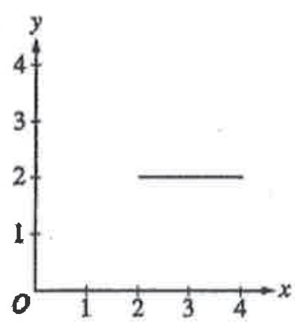
(C)



(D)



(E)



$\frac{1}{4-2}\int_2^4 f(t)\,dt = 1$

$\frac{1}{2}\int_2^4 f(t)\,dt = 1$

$\int_2^4 f(t)\,dt = 2$

2011 AP Test Free Response Question #4

NON CALCULATOR

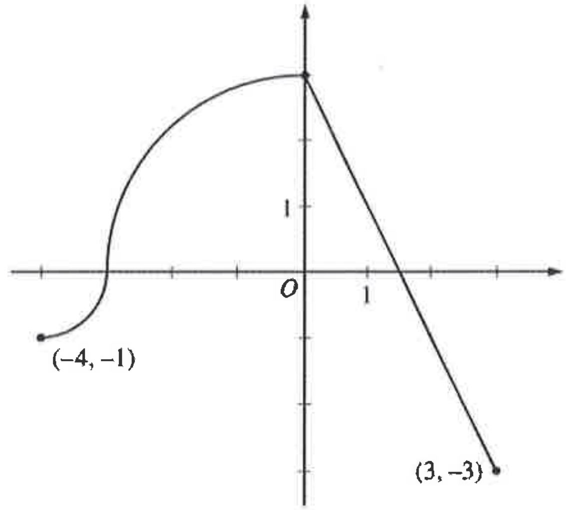
The continuous function f is defined on the interval $-4 \le x \le 3$. The graph of f consists of two quarter circles and one line segment, as shown in the figure above.

Let $g(x) = 2x + \int_0^x f(t)\,dt$.

(a) Find $g(-3)$. Find $g'(x)$ and evaluate $g'(-3)$.

(b) Determine the x-coordinate of the point at which g has an absolute maximum on the interval $-4 \le x \le 3$. Justify your answer.

(c) Find all values of x on the interval $-4 < x < 3$ for which the graph of g has a point of inflection. Give a reason for your answer.



Graph of f

(d) Find the average rate of change of f on the interval $-4 \le x \le 3$. There is no point c, $-4 < c < 3$, for which $f'(c)$ is equal to that average rate of change. Explain why this statement does not contradict the Mean Value Theorem.

a) $g(-3) = 2(-3) + \int_0^{-3} f(t)\,dt$

$= -6 - \int_{-3}^{0} f(t)\,dt$

$= -6 - \frac{1}{4}\pi(3)^2$

$= -6 - \frac{9\pi}{4}$

$g'(x) = 2 + f(x)\cdot 1$

$g'(-3) = 2 + f(-3)$

$= 2 + 0$

$= 2$

b) g has an absolute max when $g'(x)$ changes from pos. to neg. <u>or</u> at endpoints

$g'(x) = 2 + f(x) = 0$

$f(x) = -2$

$x = 2.5$

$g(2.5) > g(-4) > g(3)$ so $x = 2.5$ is where the abs. max. happens

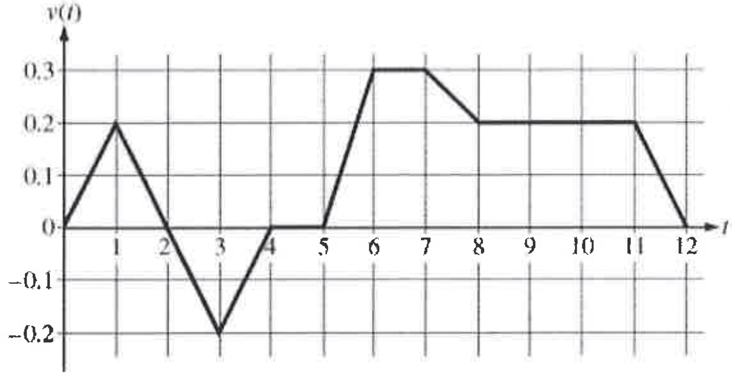
c) $g''(x) = f'(x)$ changes sign only at $x = 0$ so that is where there is a p.o.i.

d) $aROC = \frac{-3-(-1)}{3-(-4)} = \frac{-2}{7}$

$f'(c)$ doesn't need to equal $\frac{-2}{7}$ because $f(x)$ isn't differentiable on $-4 < c < 3$

2009 AP Test Free Response Question #1

CALCULATOR



Caren rides her bicycle along a straight road from home to school, starting at home at time $t = 0$ minutes and arriving at school at time $t = 12$ minutes. During the time interval $0 \le t \le 12$ minutes, her velocity $v(t)$, in miles per minute, is modeled by the piecewise-linear function whose graph is shown above.

(a) Find the acceleration of Caren's bicycle at time $t = 7.5$ minutes. Indicate units of measure.

(b) Using correct units, explain the meaning of $\int_0^{12} |v(t)|\, dt$ in terms of Caren's trip. Find the value of $\int_0^{12} |v(t)|\, dt$.

(c) Shortly after leaving home, Caren realizes she left her calculus homework at home, and she returns to get it. At what time does she turn around to go back home? Give a reason for your answer.

(d) Larry also rides his bicycle along a straight road from home to school in 12 minutes. His velocity is modeled by the function w given by $w(t) = \frac{\pi}{15}\sin\left(\frac{\pi}{12}t\right)$, where $w(t)$ is in miles per minute for $0 \le t \le 12$ minutes. Who lives closer to school: Caren or Larry? Show the work that leads to your answer.

a) $a(7.5) = \frac{-.1}{1} = -.1 \text{ mi/min}^2$

b) $\int_0^{12} |v(t)|\, dt = 1.8$ miles. This is the total distance traveled by Caren during the 12 minute trip from $t=0$ to $t=12$.

c) $t = 2$ minutes because that is when the velocity changes from pos. to neg.

d) $\int_0^{12} v(t)\, dt = 1.4$ $\int_0^{12} w(t)\, dt = 1.6$

Caren lives closer to school