Precalculus

Final Exam Review Packet

Semester 1

2017-2018

Name Key

Total _____/10 Points

Unit 1:

1. If f(x) = x^3 and g(x) = -5 $(x+2)^3$ – 6, how does the graph change from f(x) to g(x)?

① left 2
② vertical stretch
③ reflection over the x-axis
④ down 6.

2. If f(x) = $-2\sqrt{x}$ and g(x) = $-2\sqrt{x}+2$, how does the graph change from f(x) to g(x)?

up 2

3. Describe the continuity of the graph y = $\frac{x^2+9}{x+3}$. Specifically state the value of x where the discontinuity occurs.

V.A. @ $x=-3$ → infinite discontinuity

4. If f(x) = $2x^2+3$ and g(x) is the inverse of f(x), find g(x).

$g(x) = f^{-1}(x) = \pm\sqrt{\frac{x-3}{2}}$

5. Given the function $f(x) = \frac{x^2-16}{x^3-7x^2+12x}$, find any hole and horizontal and vertical asymptotes if they exit.

Hole @ $x=4$ → $(4,2)$

VA @ $x=0$, $x=3$

HA @ $y=0$

6. Prove algebraically that y = x^3 – 10 is even, odd, both or neither. Show your work.

Neither

7. Find $f(g(x))$ if $f(x) = -3x + 5$ *and* $g(x) = x^2 - 4$

$f(g(x)) = -3x^2 + 17$

8. If $f(x) = x^2$, how does the equation change with a reflection across the x-axis, a vertical translation down 2 and a horizontal translation right 3?

$T(x) = -(x-3)^2 - 2$

9. Describe the continuity of the graph $y = \frac{2}{x-3}$. Specifically state the value of x where the discontinuity occurs.

VA. @ $x = 3$ → infinite discontinuity

10. The function $f(x) = -3(x + 2)^4 + 2x^2 + 5$ has an extrema at when x = -3.
This point is considered the absolute maximum

11. State the horizontal asymptote(s), vertical asymptote(s) and holes for the function if they exist.

$h(x) = \frac{5x^2 - 7}{3x^2 + x}$

HA: $y = \frac{5}{3}$

VA: $x = 0, \; x = -1/3$

12. Prove that the function $r(x) = 5x^2 - 6$ is even and explain how you can tell from the equation and graph that it is even.

13. Prove that the function $s(x) = 3x^3 - 4x$ is odd and explain how you can tell from the equation and graph that it is odd.

Unit 2:

1. Find the zeros of each function by hand. (Hint: Factor, quadratic formula, or complete the square.)

A.) $x^2 - x - 20 = 0$

$x=5, x=-4$

B.) $3x^2 + 2x - 5 = 0$

$x = \frac{-5}{3}, x=1$

C.) $x^3 - x^2 + x - 1 = 0$

$x = \pm i, x=1$

D.) $x^4 - 256 = 0$

$x = \pm 4, x = \pm 4i$

2. Solve the inequality. State your solution graphically as well as using interval notation.

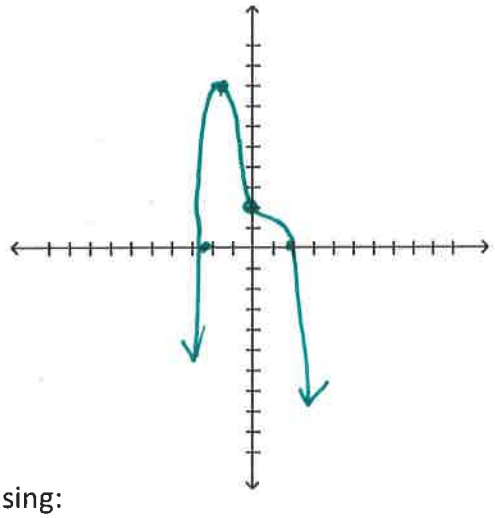
$$x^2 - 2x - 8 < 0$$

$(-2,4]$

3. Solve $\frac{x+3}{x-7} \geq 0$ using a sign chart. Be sure to write your answer in interval notation.

$(-\infty,-3] \cup (7,\infty)$

4. Use a calculator to find the following:
$y = -x^4 + 3x^2 - 3x + 2$



A.) Graph w/ zeros: (Hint: Use zero finder!)

$(-2.2, 0)$, $(1.37, 0)$

B.) Describe the end behavior:

L: As $x \to -\infty$, $y \to -\infty$

R: As $x \to \infty$, $y \to -\infty$

C.) Locate and classify extrema: (Hint: Use max/min finder!)

Abs. max: $(-1.42, 8)$

D.) Determine the intervals for which the graph is increasing and decreasing:

Increasing: $(-\infty, -1.42)$

Decreasing: $(-1.42, \infty)$

5. Solve each of the following equations. Be sure to exclude any solutions that produce a zero denominator.

A.) $n - \frac{6}{n} + 5 = 0$

$n = -6, n = 1$

B.) $\frac{-3}{x+4} + \frac{8}{x-2} = \frac{13}{x^2+2x-8}$

$x = -5$

Unit 3:

1. Write an equation of a circle with a center of (4, 6) and a radius of 6.

$(x-4)^2+(x-6)^2=36$

2. Write an equation of a circle that has its center at (3, -2) and area $20\pi\ in^2$.

$(x-3)^2+(y+2)^2=20$

3. Identify the center, foci, vertices, co-vertices and lengths of major and minor axis of the ellipse. Then, graph it on the coordinate plane.

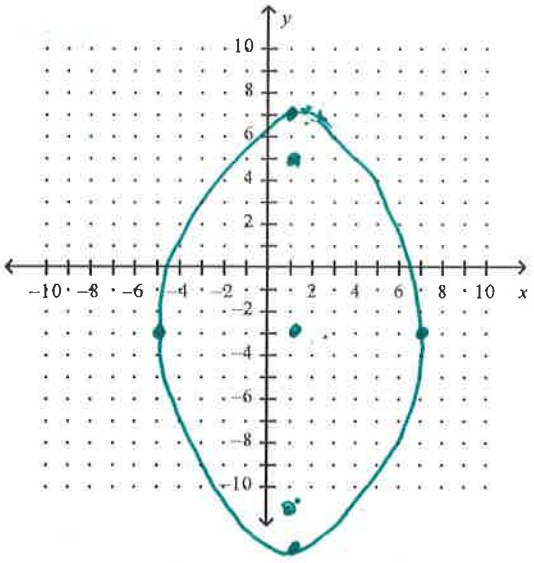
$\frac{(y+3)^2}{100}+\frac{(x-1)^2}{36}=1$

Center: $(1,-3)$

Vertices: $(1,7),(1,-13)$

co-vertices: $(7,-3),(-5,-3)$

Foci : $(1,5),(1,-11)$



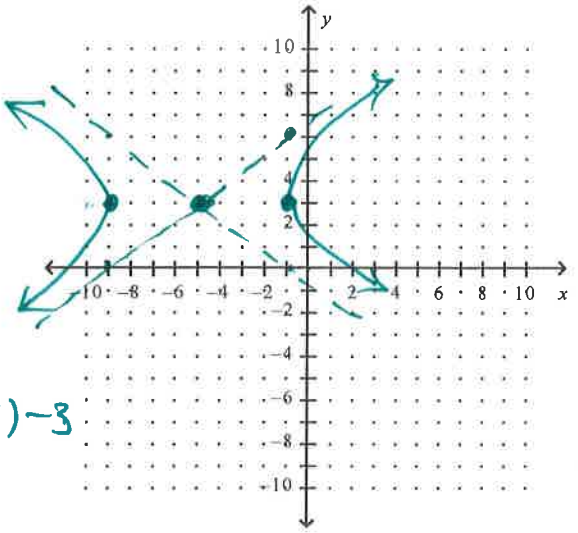
4. Identify the center, vertices, foci and slopes of the asymptotes of the hyperbola. Then, graph it on the coordinate plane.

$\frac{(x+5)^2}{16}-\frac{(y-3)^2}{9}=1.$

Center: $(-5,3)$

Foci : $(-10,3),(-1,3)$

Asymptotes: $y=\pm\frac{3}{4}(x+5)-3$



5. Identify the vertex, focus and directrix of the parabola. Then, graph it by plotting three accurate points.

$(x+3)^2 = -12(y-2)$

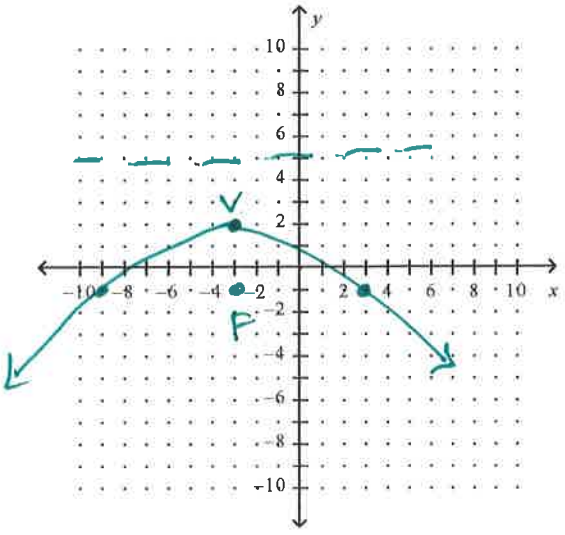
Vertices: $(-3, 2)$

$p = -3$

Focus $= (-3, -1)$

Directrix: $y = 5$

Focal width $= 12$



6. Identify the type of conic. Then, convert it to standard form.

$3y^2 - 4x^2 - 8x - 18y + 19 = 0$

$$\frac{3(y-3)^2}{4} - \frac{(x+1)^2}{4} = 1$$

7. Identify the type of conic. Then, convert it to standard form.

$x^2 + 2x + 8y - 7 = 0$

$$(x+1)^2 = -8(y-1)$$

Unit 4:

1. Convert to log form: $x^2 = 49$.

$\log_x 49 = 2$

2. State the asymptote of $y = 8e^{x-7} - 12$

$y = -12$

3. Approximately how long will it take \$20,000 to double if it is invested at an interest rate of 9% that is compounded quarterly?

$t = 7.79$ years

4. Condense $\log_8 3 - 3\log_8 x + \log_8 w$

$\log_8\left(\frac{3w}{x^3}\right)$

5. Solve $\log_{10} x^2 - \log_{10} 4 = \log_{10} 16$

$x = \pm 8$

6. Solve $10^{x+1} = 502$ to the nearest hundredth.

$x = 1.70$

7. Simplify the following exponential expression: $\left(\frac{16x^9y^8}{2x^3y^5}\right)^{\frac{1}{3}}$

$2x^2y$

8. Find the balance in an account at the end of 23 years if $6,500 is invested at an interest rate of 3.2% that is compounded:

A.) monthly B.) quarterly C.) continuously

$A = 13,555.91$ $A = 13,529.91$ $A = 13569.20$

9. Write $3^x = 8$ in logarithmic form.

$\log_3 8 = x$

10. Write $\log_2 8 = 3$ in exponential form.

$2^3 = 8$

11. Solve $e^{5x-3} = 20$

$x = 1.2$

12. Solve $\log_{13} x^3 - \log_{13} 2 = \log_{13} 6$.

$x = \sqrt[3]{12}$

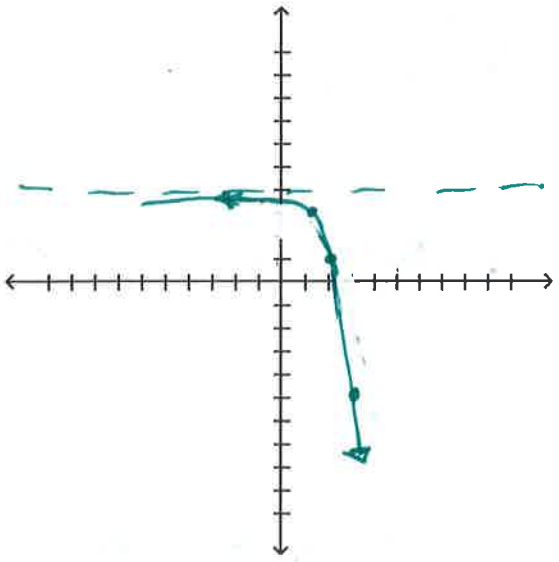
13. Solve log3 + log x = log 21

$x = 7$

14. Solve $7^{x+8} = 2$

$x = -7.64$

15. Sketch a graph of $f(x) = -(3)^{x-1} + 4$ and label the following information:



Domain: $(-\infty, \infty)$

Range: $(-\infty, 4)$

Asymptote: $y = 4$

End behavior:

L: As $x \to -\infty$, $y \to 4$

R: As $x \to \infty$; $y \to -\infty$

Unit 5:

1. Write a general term formula for a sequence where $a_1 = 200$ and $r = \frac{1}{2}$ and then find the 20^{th} term.

$$a_{20} = 3.81 \times 10^{-4}$$

2. Given the sequence: $8 + 1 + -6 + -13 + \ldots$

   A.) Find the formula of the general term.

$$a_n = -7n + 15$$

   B.) Find the 50^{th} term.

$$a_{50} = -335$$

3. If the 33^{rd} term in an arithmetic sequence is 19 and the 34^{th} term is 22, find the first term.

$$a_1 = -77$$

4. Express each of the following series in Sigma notation.

   A.) $160 + 40 + 10 + \ldots$

$$\sum_{n=1}^{\infty} 160\left(\frac{1}{4}\right)^{n-1}$$

   B.) $7 + 10 + 13 + \ldots + 43$

$$\sum_{n=1}^{13} 3n + 4$$

5. Find the sum of each of the following series. Write one sentence explaining the solution:

A.) $100 + 50 + 25 + \ldots + 3.125$.

$S_6 = 196.875$

B.) $8 + 6 + 4 + \ldots$

the sum does not exist because it is diverging to $-\infty$

6. Evaluate each of the following sums:

A.) $\sum_{n=8}^{10} 2n - 1 = 51$

B.) $\sum_{n=1}^{3} 3n + 5 = 33$

C.) $\sum_{n=0}^{2} 6\left(\frac{2}{3}\right)^{n-1} = 19$

7. Evaluate each of the following limits algebraically:

A.) $\lim_{x\to 5} \frac{x^2-6x+5}{x-5} = 4$

B.) $\lim_{x\to 1} 3x + 2 = \lim_{x\to 4} \frac{x^2-5x+6}{x-2} = 1$

C.) $\lim_{x\to 1} \frac{x-1}{x^2-4x+3} = -\frac{1}{2}$

D.) $\lim_{x\to 3} \frac{x^2-9}{x^2-4x+3} = 3$