Due: May 4th, 2018

Name Answer key

#5

AP Calculus Problem Set
AP Test Preparation

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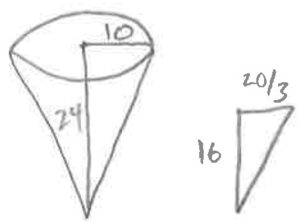
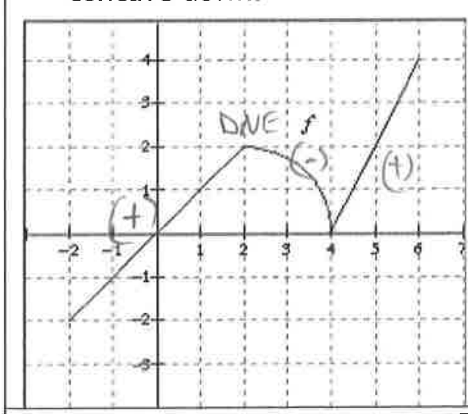
Directions: Show work for every problem. No work = no credit. You can attach your work if you need more room but your final answers need to go in the boxes below. This is **NON-CALCULATOR** unless otherwise specified.

Problem	Final Answer
1. Antiderive: $\int \cos^3 x \sin x\, dx$ let $u = \cos x$ $du = -\sin x\, dx$ $-\int u^3 du = -\frac{u^4}{4} + c$ $-\frac{\cos^4 x}{4} + c$	$\boxed{-\frac{\cos^4 x}{4} + c}$
2. A conical water tank with vertex down has a radius of 10 feet at the top and is 24 feet high. If water flows out of the tank at rate of 20 ft³/min, how fast is the depth of the water decreasing when the water is 16 feet deep?  $\frac{24}{10} = \frac{h}{r}$ $r = \frac{5}{12}h$ $V = \frac{1}{3}\pi r^2 h$ $V = \frac{1}{3}\pi\left(\frac{5}{12}h\right)^2 \cdot h$ $V = \frac{25\pi}{432}h^3$ $\frac{dV}{dt} = \frac{25\pi}{144}h^2\frac{dh}{dt}$ $-20 = \frac{400\pi}{9}\frac{dh}{dt}$ $\frac{dh}{dt} = -\frac{9}{20\pi}$	$\boxed{-\frac{9}{20\pi} \text{ ft/min}}$
3. Let $F(x) = \int_2^x f(t)\, dt$. The graph of f on the interval $[-2, 6]$ consists of two line segments and a quarter of a circle. Determine $F(0)$, any x-coordinates of points of inflection of $F(x)$ and the interval(s) where $F(x)$ is concave down.  $F(x) = -\int_x^2 f(t)\,dt$ $F(0) = -\int_0^2 f(t)\,dt$ $F(0) = -2$ $F'(x) = f(x)$ $F''(x) = f'(x)$	$\boxed{F(0) = -2}$ POI of $F(x)$ at $x = 2, 4$ $F(x)$ is CD over $(2, 4)$
4. If $g(x) = f^{-1}(x)$, and $f(x) = \sqrt{x-4}$ then solve for $g'(1)$. $f'(x) = \frac{1}{2\sqrt{x-4}}$ $g'(1) = \frac{d}{dx} f^{-1}(1) = \frac{1}{f'(f^{-1}(1))} = \frac{1}{f'(5)} = \frac{1}{1/2(1)} = 2$	$\boxed{2}$

5. Find $\frac{dy}{dx}$ given $y = \left(\frac{x+3}{x^2+6}\right)^8$

$$\frac{dy}{dx} = 8\left(\frac{x+3}{x^2+6}\right)^7 \cdot \frac{(x^2+6)\cdot 1 - (x+3)(2x)}{(x^2+6)^2}$$

Answer: $\boxed{= 8\left(\frac{x+3}{x^2+6}\right)^7 \cdot \frac{-x^2-6x+6}{(x^2+6)^2}}$

6. The figure below shows the graph of $y = 5x - x^2$ and the graph of the line $y = 2x$. What is the area of the shaded region?

$$A = \int_0^3 (5x - x^2 - 2x)\,dx \quad \text{or} \quad \int_0^3 (-x^2+3x)\,dx$$

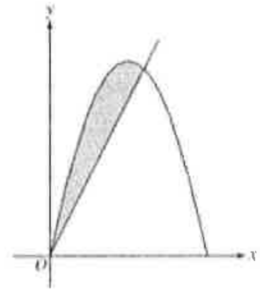
$$-\frac{x^3}{3} + \frac{3x^2}{2}\Big|_0^3$$

$$-9 + \frac{27}{2} - (0)$$

limits

$2x = 5x - x^2 \Rightarrow 0 = x(3-x)$

$0 = 3x - x^2$ $\quad x = 0, 3$



Answer: $\boxed{\frac{9}{2}}$

7. Find $\frac{dy}{dx}$ given $y = \arcsin\sqrt{x}$ or $y = \sin^{-1}(\sqrt{x})$

$$\frac{d}{dx} = \frac{1}{\sqrt{1-(\sqrt{x})^2}} \cdot \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}\cdot\sqrt{1-x}}$$

Answer: $\boxed{\frac{1}{2\sqrt{x}\cdot\sqrt{1-x}}}$

8. Find $\lim\limits_{t\to1} \frac{5t^4-4t^2-1}{10-t-9t^3} = \frac{5-4-1}{10-1-9} = \frac{0}{0}$ l'hospital's

$$\lim_{t\to1} \frac{20t^3-8t}{-1-27t^2} = \frac{20-8}{-1-27} = \frac{12}{-28} = -\frac{3}{7}$$

Answer: $\boxed{-\frac{3}{7}}$

9. Find the average value and the average rate of change of $3x^2 - 2x$ on [1, 4].

$$\frac{1}{4-1}\int_1^4 (3x^2-2x)\,dx$$

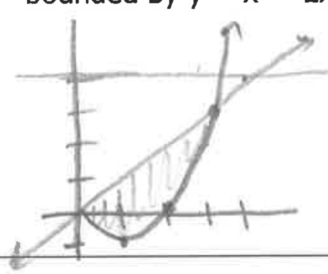
$$= \frac{1}{3}\left[x^3 - x^2\right]_1^4 = \frac{1}{3}\left[48 - 0\right] = 16$$

$$\frac{f(4)-f(1)}{4-1} = \frac{40-1}{3} = 13$$

Average value: $\boxed{16}$

Average rate of change: $\boxed{13}$

10. Determine the volume of the solid obtained by rotating the region bounded by $y = x^2 - 2x$ and $y = x$ about the line $y = 4$. [CALC]



$$V = \pi\int_0^3 \left((4-(x^2-2x))^2 - (4-x)^2\right)dx$$

Answer: $\boxed{96.132}$ or $\boxed{96.133}$