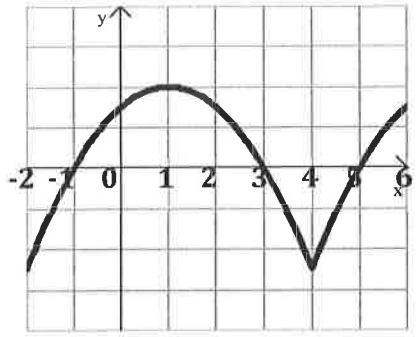
Due: April 27, 2018 Name Answer key

#4

AP Calculus Problem Set
AP Test Preparation

10

Directions: Show work for every problem. No work = no credit. You can attach your work if you need more room but your final answers need to go in the boxes below.

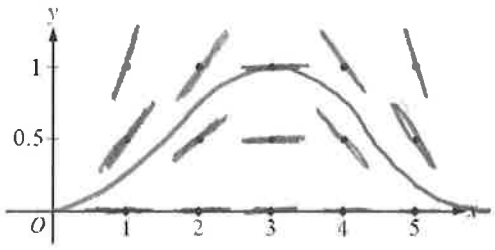
Problem	Final Answer
1. Below is a graph of f', the derivative of f. Determine all intervals of f that are concave up, concave down, increasing and decreasing. 	Inc: $(-1,3) \cup (5,6)$ Dec: $(-2,-1) \cup (3,5)$ CU: $(-2,1) \cup (4,6)$ cd: $(1,4)$
2. Given the function $f(x) = 5 - 12x + 3x^2$, find the value of x within the interval from $[1,3]$ that guarantees that the instantaneous rate of change equals the average rate of change. $\frac{f(3)-f(1)}{3-1} = \frac{-4-(-4)}{2} = \frac{0}{2} = 0$ $6x-12=0$ $6x=12$ $x=2$	$x=2$
3. Determine the intervals of time in which speed is decreasing given the function: $x(t) = t^3 - 4t^2 - 3t - 1$ and $t \geq 0$. $v(t) = 3t^2 - 8t - 3$ $0 = 3t^2 - 8t - 3$ $0 = (3t+1)(t-3)$ $t = -\frac{1}{3}, 3$ $a(t) = 6t - 8$ $8 = 6t$ $t = 4/3$ (+) (−) a(t) 0 4/3 (−) (+) v(t) 0 3	$(\frac{4}{3}, 3)$
4. Given the velocity function, $v(t) = -2t + 4$, calculate the total distance and the displacement of the function from $t = -2$ to $t = 5$. disp: $\int_{-2}^{5} (-2t+4)\,dt$ $-t^2+4t\Big|_{-2}^{5}$ $= -5-(-12) = 7$ $0 = -2t+4$ $t = 2$ changes dir @ TD: $-t^2+4t\Big|_{-2}^{2} + -t^2+4t\Big|_{2}^{5}$ $\lvert 4-(-12)\rvert + \lvert -5+4\rvert$ $\lvert 16\rvert + \lvert -9\rvert = 25$	Disp: 7 TD: 25

5. Find the volume of the solid that results when the region enclosed by the curves $y = 2x + 2$ and $y = x^2 + 2$ is revolved about the line $y = -1$. [CALC]

$$\pi \int_0^2 \left[(2x+3)^2 - (x^2+3)^2\right] dx$$

38.537
38.536

6. Sketch a slope field for the differential equation: $\frac{dy}{dx} = y^2(6 - 2x)$ on the axis provided. Then draw a particular solution with the initial condition $f(3) = 1$.



7. Solve for $\frac{dy}{dx}$ if $2y^3 + 4x^2 - y = 6$

$$6y^2 \frac{dy}{dx} + 8x - \frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(6y^2 - 1) = -8x$$

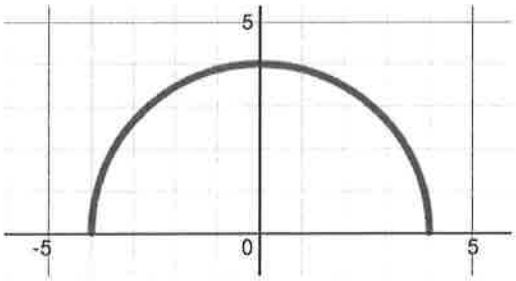
$$\frac{dy}{dx} = \frac{-8x}{6y^2 - 1}$$

8. Given the equation $y = \sqrt{x-1}$, find an equation of the normal line to the graph at $x = 5$. (5, 2) $m = \frac{1}{4}$

$y' = \frac{1}{2}(x-1)^{-1/2} = \frac{1}{2\sqrt{x-1}}$

$y - 2 = -4(x - 5)$

9. Find f(-4) given that f(4) = 7 and the graph of f' is given below.



$$\int_{-4}^{4} f'(x) = f(4) - f(-4)$$

$8\pi = 7 - f(-4)$

$f(-4) = 7 - 8\pi$

$7 - 8\pi$

10. A manufacturer wants to design an open box having a square base and a surface area of 108 square inches. What dimensions will produce a box with maximum volume?

$108 = x^2 + 4xy \iff y = \frac{108 - x^2}{4x}$

$V = x^2 y$

$V = x^2\left(\frac{108 - x^2}{4x}\right)$

$V = 27x - \frac{1}{4}x^3$

$108 = 36 + 24y$

$y = 3$

$0 = 27 - \frac{3}{4}x^2 \rightarrow 36 = x^2 \rightarrow x = 6$

$x = 6$
$y = 3$