Due: April 20, 2018

Name Answer key

#3

AP Calculus Problem Set
AP Test Preparation

___ 10

Directions: Show work for every problem. No work = no credit. You can attach your work if you need more room but your final answers need to go in the boxes below.
This worksheet is NON-CALCULATOR unless otherwise specified

Problem	Final Answer

1. The values for a continuous function f(x) are given in the table below. What is the trapezoidal rule approximation for $\int_0^8 f(x)\,dx$ with 8 sub-intervals? What is the midpoint approximation for $\int_0^8 f(x)\,dx$ with 4 sub-intervals?

x	0	1	2	3	4	5	6	7	8
f(x)	15	12.5	6	-3	-5	2	3.5	7.5	10

Trap $= \frac{1}{2}(1)(15+2(12.5)+2(6)+2(-3)+2(-5)+2(2)+2(3.5)+2(7.5)+10)$
$= \frac{1}{2}(40+6+1+25) = \frac{1}{2}(72) = 36$

Mid $= 2(12.5)+2(-3)+2(2)+2(7.5) = 19+19 = 38$

Final Answer: Trap: $\boxed{36}$ mid: $\boxed{38}$

2. Using the values below, evaluate $q'(4)$ if $q(x) = \frac{r(x)}{s(x)}$.

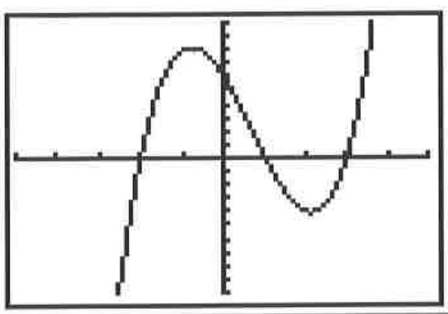
$r(4) = 2 \quad r(2) = 0 \quad r'(4) = -1 \quad s(4) = 3 \quad s'(4) = 1 \quad s'(2) = 4$

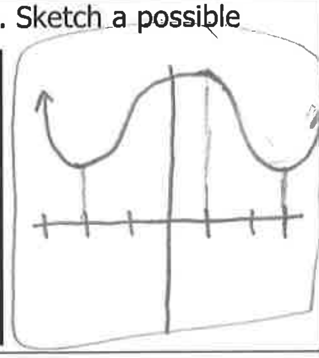
$q'(x) = \dfrac{s(x)\cdot r'(x) - r(x)\cdot s'(x)}{[s(x)]^2}$

$q'(4) = \dfrac{3(-1) - 2(1)}{3^2} = -5/9$

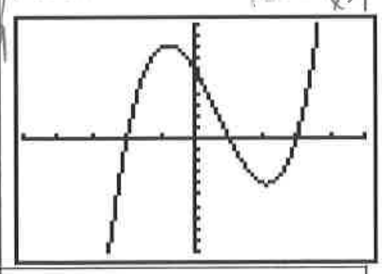
Final Answer: $q'(4) = \boxed{-\frac{5}{9}}$

3. The graph below is the derivative of the graph of f(x). Sketch a possible graph of f(x).





Final Answer: Minimum's at x = -2, 3
Maximum at x = 1
POI's at x = -1, 2



4. The position of a particle traveling along a straight line is $x(t) = t^3 - 9t^2 + 15t + 3$. On the interval t = 0 to t = 10, when is the particle farthest to the left? → x(t) is at a minimum

$v(t) = 3t^2 - 18t + 15$

$0 = t^2 - 6t + 5$

$0 = (t-5)(t-1)$

$t = 1, 5$

EVT

t	x(t)
0	3
1	10
5	-22
10	253

$125 - 225 + 75 + 3$
$= -100 + 78$
$= -22$

$1000 - 900 + 150 + 3$
253

Final Answer: $\boxed{t = 5}$

5. What is $\lim_{h\to 0} \frac{\csc\left(\frac{\pi}{4}+h\right)-\csc\left(\frac{\pi}{4}\right)}{h}$?

$= -\csc\frac{\pi}{4}\cot\frac{\pi}{4}$

$= -\sqrt{2}\cdot 1$

$\boxed{-\sqrt{2}}$

6. If f(x) = $x^2 - 5x + 3$, show that there must exist a point *c* on the interval (0, 4) such that f(c) = 1. Then, find the value of c. What theorem is this? [CALC]

$f(4) = 4^2 - 5(4) + 3 = -1$

$f(0) = 0^2 - 5(0) + 3 = 3$

$1 = x^2 - 5x + 3$

$0 = x^2 - 5x + 2$

$x = \frac{5 \pm \sqrt{25-8}}{2}$

$x = \frac{5 \pm \sqrt{17}}{2}$

$-1 < 1 < 3$, IVT states there is a value c where $f(c) = 1$

$\boxed{x = \frac{5-\sqrt{17}}{2}}$

$\boxed{\text{IVT}}$

7. Given the function f(x) = $x^4 - 4x^2$, how many relative minimums does it have? How many relative maximums? How many points of inflection?

$f'(x) = 4x^3 - 8x$

$0 = 4x^3 - 8x$

$0 = 4x(x^2 - 2)$

$x = 0, \pm\sqrt{2}$

(−) (+) (−) (+)

$-\sqrt{2}$ 0 $\sqrt{2}$

$f''(x) = 12x^2 - 8$

$0 = 12x^2 - 8$

$x^2 = \frac{2}{3}$

$x = \pm\sqrt{\frac{2}{3}} = \pm\frac{\sqrt{6}}{3}$

(+) (−) (+)

$-\frac{\sqrt{6}}{3}$ $\frac{\sqrt{6}}{3}$

Minimums: 2

Maximums: 1

POI's: 2

8. Find the values of a and b that make f continuous everywhere.

$$f(x) = \begin{cases} x^2 - 4x - 2 & \text{if } x < 2 \\ ax^2 - bx + 3 & \text{if } 2 \le x < 3 \\ 2x - a + b & \text{if } x \ge 3 \end{cases}$$

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$

$2^2 - 4(2) - 2 = a(2)^2 - b(2) + 3$

$-6 = 4a - 2b + 3$

$4a - 2b = -9$ mult by −2

$10a - 4b = 3$

$-8a + 4b = 18$

$2a = 21$

$a = \frac{21}{2}$

$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x)$

$9a - 3b + 3 = 6 - a + b$

$10a - 4b = 3$

$4\left(\frac{21}{2}\right) - 2b = -9$

$42 - 2b = -9$

$-2b = -51$

$b = \frac{51}{2}$

$a = \frac{21}{2}$

$b = \frac{51}{2}$

9. Is f′(c) = 0 guaranteed on the interval [-1, 4] when f(x) = $x^2 - 3x - 4$? If so, what is the value of c? If not, why not? What theorem does the correlate to?

Secant slope over $[-1, 4] = \frac{f(4) - f(-1)}{4-(-1)} = \frac{0-0}{5} = 0$

$f'(c) = 2c - 3$

$0 = 2c - 3$

$c = 3/2$

yes, $f'(c) = 0$ over the interval by the MVT

$\boxed{c = \frac{3}{2}}$

10. Find the absolute extreme values of the function f(x) = $\frac{x}{x^2+1}$ on the interval [-3, 3].

$f'(x) = \frac{(x^2+1)\cdot 1 - x(2x)}{(x^2+1)^2}$

$0 = -x^2 + 1$

$x^2 = 1$ $x = \pm 1$

EVT

x	f(x)
−3	−3/10
−1	−1/2
1	1/2
3	3/10

Maximum value of 1/2

Minimum value of −1/2