Due: April 13, 2018

Name Answer key

#2

AP Calculus Problem Set
AP Test Preparation

10

Directions: Show work for every problem. No work = no credit. You can attach your work if you need more room but your final answers need to go in the boxes below.

Problem | **Final Answer**

1. Find the x-value in the interval [0, 2] where the instantaneous rate of change of $r(x) = x^3 - x$ equals the average rate of change between the two endpoints. What theorem is this?

$r'(x)$

$\hookrightarrow 3x^2 - 1 = \frac{r(2) - r(0)}{2 - 0}$

$3x^2 - 1 = 3$

$3x^2 = 4$

$x^2 = \frac{4}{3}$

$x = \frac{2}{\sqrt{3}}$ or $\frac{2\sqrt{3}}{3}$

Final Answer: $\boxed{x = \frac{2\sqrt{3}}{3}}$

$\boxed{\text{MVT}}$

2. Given the table below of values of the differentiable functions f(x) and g(x), if h(x) = f(g(x)), find h′(2).

x	f(x)	f'(x)	g(x)	g'(x)
-4	12	10	5	-3
2	5	-8	-4	2
5	2	-3	0	5

$h'(x) = f'(g(x)) \cdot g'(x)$

$h'(2) = f'(g(2)) \cdot g'(2)$

$= f'(-4) \cdot 2$

$= 10 \cdot 2$

$= 20$

Final Answer: $\boxed{20}$

3. If g(x) = $\int_0^{x^2} \sin(t^3)\, dt$, find g′(x) and g″(x).

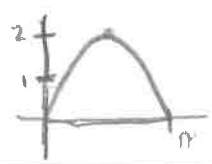
$g'(x) = \sin(x^6) \cdot 2x$

$g''(x) = \cos(x^6) \cdot 6x^5 \cdot 2x + 2\sin(x^6)$

Final Answer: $\boxed{g'(x) = 2x \sin(x^6)}$

$\boxed{g''(x) = 12x^6 \cos(x^6) + 2\sin(x^6)}$

4. A mathematician has a paperweight made so that its base is the shape of the region between the x-axis and one arch of the curve y = 2sinx. Each cross-section perpendicular to the x-axis is an equilateral triangle whose diameter runs from the x-axis to the curve. Find the volume of the paperweight. [CALC]



$V = \frac{\sqrt{3}}{4} \int_0^{\pi} [2\sin x]^2\, dx$

Final Answer: $\boxed{2.720}$

or

2.721

5. Find the general solution if $\frac{dy}{dx} = \frac{2x-x^2}{3y}$ $\int 3y\,dy = \int (2x-x^2)\,dx$ $\frac{3y^2}{2} = x^2 - \frac{x^3}{3} + c$ $y^2 = \frac{2}{3}x^2 - \frac{2x^3}{9} + \frac{2}{3}c$ y	$\boxed{y = \pm\sqrt{\frac{2}{3}x^2 - \frac{2x^3}{9} + \frac{2}{3}c}}$
6. What is the average value of the function g(x) = $(2x + 3)^2$ on the interval [-3, -1]? $= \frac{1}{-1-(-3)} \int_{-3}^{-1} (2x+3)^2\,dx$ $= \frac{1}{2}\left[\frac{(2x+3)^3}{6}\right]_{-3}^{-1}$ $= \frac{1}{2}\left[\frac{(-2+3)^3}{6} - \frac{(-6+3)^3}{6}\right]$ $= \frac{1}{2}\left[\frac{1}{6} + \frac{27}{6}\right]$ $= \frac{1}{2}\left[\frac{28}{6}\right]$	$\boxed{\frac{7}{3}}$
7. What is the value of $\int_1^{e^2} \frac{1}{x}\,dx$? $\ln\|x\|\Big\|_1^{e^2} = \ln\|e^2\| - \ln\|1\|$ $= 2\ln e - 0$ $= 2$	$\boxed{2}$
8. What is the value of $\lim_{x\to\infty} \frac{x^2-5x+6}{x-3}$?	$\boxed{\text{DNE}}$ or ∞
$4(\cos x)^2$ 9. Determine the concavity of the graph of f(x) = 3sinx + 4 $\cos^2$x at x = π. $f'(x) = 3\cos x + 8\cos x \cdot -\sin x$ $f''(x) = -3\sin x + 8\cos x \cdot -\cos x + -\sin x \cdot 8\sin x$ $f''(x) = -3\sin x - 8(\cos^2 x + \sin^2 x)$ $\quad f''(\pi) = -8$	$\boxed{\text{f(x) is cd at } x=\pi}$
10. A particle's acceleration for t $\geq$ 0 is given by a(t) = 12t + 4. The particle's initial position is 2 and its velocity at t = 1 is 5. What is the position of the particle at t = 2? $\int 12t+4 = 6t^2+4t+C$ $5 = 6+4+C$ $-5 = C$ $v(t) = 6t^2+4t-5$ $\int 6t^2+4t-5$ $= 2t^3+2t^2-5t+c$ $2 = 2(0)^3+2(0)^2-5(0)+c$ $c = 2$ $p(2) = 2(2)^3+2(2)^2-5(2)+2$ $= 16+8-10+2$ $= 14+2 = 16$	$\boxed{16}$