Due: April 6, 2018 Name Answer key

#1 AP Calculus Problem Set

AP Test Preparation

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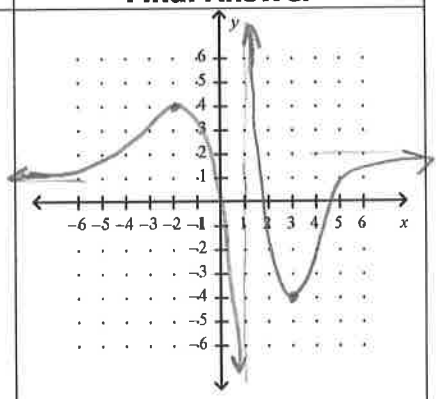
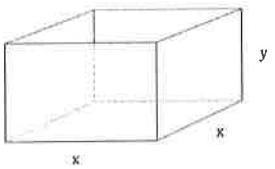
Problem	Final Answer
1. Sketch a graph that meets the following limit conditions: $\lim_{x\to\infty} f(x) = 2$ $\lim_{x\to-\infty} f(x) = 1$ $\lim_{x\to3} f(x) = -4$ $\lim_{x\to1^+} f(x) = \infty$ $\lim_{x\to1^-} f(x) = -\infty$ $\lim_{x\to-2} f(x) = 4$	
2. Given $x^2 + 3y^2 = 4y$, use implicit differentiation to find $\frac{dy}{dx}$. $2x + 6y\frac{dy}{dx} = 4\frac{dy}{dx}$ $6y\frac{dy}{dx} - 4\frac{dy}{dx} = -2x$ $\frac{dy}{dx}(6y-4) = \frac{-2x}{6y-4}$	$\frac{dy}{dx} = \frac{-2x}{6y-4}$
3. Find the largest possible volume for an open-top rectangular box with a square base and surface area of 48 in^2.  $SA = x^2 + 4xy$ $48 = x^2 + 4xy$ $\frac{48-x^2}{4x} = y$ $V = x^2y$ $V = x^2\left(\frac{48-x^2}{4x}\right)$ $V = 12x - \frac{x^3}{4}$ $\frac{dV}{dx} = 12 - \frac{3x^2}{4}$ $0 = 12 - \frac{3x^2}{4} \Rightarrow 3x^2 = 48 \Rightarrow x^2 = 16$	$V = 4^2 \cdot 2$ $V = 16 \cdot 2$ $V = 32\text{ in}^3$
4. Find the velocity of the particle at the times when the acceleration is zero given the position function: $s(t) = t^3 - 3t^2 + 8t + 2$ $v(t) = 3t^2 - 6t + 8$ $a(t) = 6t - 6$ $0 = 6t - 6$ $t = 1$ $v(1) = 3(1)^2 - 6(1) + 8$ $= 3 - 6 + 8$ $= 5$	$v(1) = 5$

5. If $f(x) = x\sqrt{2x-3}$, then $f'(x) =$

$f'(x) = x \frac{d}{dx}(2x-3)^{1/2} + \sqrt{2x-3} \cdot 1$

$f'(x) = x \cdot \frac{1}{2}(2x-3)^{-1/2} \cdot 2 + \sqrt{2x-3}$

$= \frac{x}{\sqrt{2x-3}} + \frac{\sqrt{2x-3}}{1} \cdot \frac{\sqrt{2x-3}}{\sqrt{2x-3}}$

$= \frac{3x-3}{\sqrt{2x-3}}$

$f'(x) = \frac{3x-3}{\sqrt{2x-3}}$

6. Write the equation of the tangent line at the point where $x = 1$ for the equation: $y = 2x^3 - x^2 + 1$:

$y = 2(1)^3 - 1^2 + 1$

$y = 2 - 1 + 1$

$y = 2$

$\frac{dy}{dx} = 6x^2 - 2x$

$\frac{dy}{dx}\Big|_{x=1} = 6(1)^2 - 2(1) = 6 - 2 = 4$

$y - 2 = 4(x-1)$

7. Given that $h(x) = f^{-1}(x)$, find $h'(4)$.

x	2	3	4	5
f(x)	5	4	3	2
f'(x)	1	2	3	4

$h'(4) = \frac{1}{f'(f^{-1}(4))}$

$= \frac{1}{f'(3)} = \frac{1}{2}$

$\frac{1}{2}$

8. A spherical balloon is inflated at $12\pi\, ft^3/min$. Find the rate of change of the radius of the balloon at the instant when the volume is $\frac{32}{3}\pi\, ft^3$. (Hint: The volume of a balloon is given by $V = \frac{4}{3}\pi r^3$)

$32 = 4r^3$

$8 = r^3$

$r = 2$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$12\pi = 4\pi(4)\frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{3}{4}\, ft/min$

$\frac{dr}{dt} = \frac{3}{4}\, ft/min$

9. Find $\lim_{x\to\infty} \frac{-2x^3+x}{-4x^5+2x^2+2}$. Explain your answer.

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10. Given $5x^3 - 4xy - 2y^2 = 1$ find dy/dx.

$15x^2 - 4x\frac{dy}{dx} + y(-4) - 4y\frac{dy}{dx} = 0$

$-4x\frac{dy}{dx} - 4y\frac{dy}{dx} = 4y - 15x^2$

$\frac{dy}{dx} = \frac{4y - 15x^2}{-4x - 4y}$