

Inverses of Rational Functions

Determine the inverse of each of the following functions:

1. $f(x) = \frac{5}{x-4}$

$$y = \frac{5}{x-4}$$

$$(y-4)x = \frac{5}{(y-4)}(y-4)$$

$$(y-4)x = \frac{5}{x}$$

$$y-4 = \frac{5}{x} + 4$$

$$f^{-1}(x) = \frac{5}{x} + 4$$

or $= \frac{4x+5}{x}$

2. $g(x) = \frac{x-7}{2x}$

$$y = \frac{x-7}{2x}$$

$$2yx = \frac{(y-7)}{2y} \cdot 2y$$

$$2yx = y-7$$

$$2yx - y = -7$$

$$y(2x-1) = -7$$

$$g^{-1}(x) = \frac{-7}{2x-1}$$

3. $h(x) = \frac{3x+1}{x-1}$

$$(y-1)x = \frac{(3y+1)}{(y-1)}(y-1)$$

$$yx - x = 3y + 1$$

$$yx - 3y = x + 1$$

$$y(x-3) = x+1$$

$$h^{-1}(x) = \frac{x+1}{x-3}$$

4. $k(x) = \frac{x+4}{2x-3}$

$$(2y-3)x = \frac{(y+4)}{(2y-3)}(2y-3)$$

$$2yx - 3x = y + 4$$

$$2yx - y = 3x + 4$$

$$y(2x-1) = 3x + 4$$

$$k^{-1}(x) = \frac{3x+4}{2x-1}$$