Integrated Math 3
Unit 7: Modeling Rational Representations
7.4

Name: Completed Nots

Date:__________ Period:______

Objective: To determine the inverse of a rational function.

Recall:

An **inverse** function has the unique property of mirroring a function over the line $y = x$.

How to Determine an Inverse Function

- Change the function notation to $y =$
- Switch x and y
- Solve for y (get y by itself)
- Rewrite y using $f^{-1}(x)$

Example 1: $f(x) = \frac{3}{x+2}$

$$y = \frac{3}{x+2}$$

$$(y+2)x = \frac{3}{\cancel{(y+2)}} \cdot \cancel{(y+2)}$$

$$\frac{(y+2)x}{x} = \frac{3}{x}$$

$$y + \cancel{2} = \frac{3}{x} - 2$$
$$\cancel{-2}$$

$$y = \frac{3}{x} - 2$$

$$\boxed{f^{-1}(x) = \frac{3}{x} - 2}$$

or

$$(y+2)x = \frac{3}{\cancel{(y+2)}} \cdot (y+2)$$

$$yx + \cancel{2x} = 3 - 2x$$
$$\cancel{-2x}$$

$$\frac{yx}{x} = \frac{3-2x}{x}$$

$$y = \frac{3-2x}{x}$$

$$\boxed{f^{-1}(x) = \frac{3-2x}{x}}$$

*these are actually the same!

Example 2: $g(x) = \frac{x+4}{x}$

$$y = \frac{x+4}{x}$$

$$y\,x = \frac{y+4}{\cancel{y}}\,\cancel{y}$$

$$yx = y + 4$$
$$-y \quad -y$$

$$yx - y = 4$$

$$\frac{y\cancel{(x-1)}}{\cancel{(x-1)}} = \frac{4}{(x-1)}$$

$$y = \frac{4}{x-1} \qquad \boxed{f^{-1}(x) = \frac{4}{x-1}}$$

Example 3: $h(x) = \frac{2x-5}{x+3}$

$$y = \frac{2x-5}{x+3}$$

$$(y+3)x = \frac{2y-5}{\cancel{(y+3)}}\,\cancel{(y+3)}$$

$$-2y + yx \cancel{+3x} = 2y - 5 - 3x$$
$$\cancel{-3x} \quad -2y$$

$$-2y + yx = -5 - 3x$$

$$\frac{y(-2+x)}{(-2+x)} = \frac{-5-3x}{(x-2)}$$

$$y = \frac{-5-3x}{x-2} \text{ or } -\frac{3x+5}{x-2} \qquad \boxed{f^{-1}(x) = -\frac{3x+5}{x-2}}$$