

Objective: To add and subtract rational expressions with like and unlike denominators.

Warm Up: Find the sum or difference of the following fractions.

a) $\frac{1}{5} + \frac{2}{5} = \frac{1+2}{5} = \boxed{\frac{3}{5}}$

b) $\frac{1}{3} + \frac{2}{4} = \boxed{\frac{5}{6}}$

$$\frac{1}{3} \cdot \frac{4}{4} + \frac{2}{4} \cdot \frac{3}{3} = \frac{4}{12} + \frac{6}{12} = \frac{10}{12} = \frac{5}{6}$$

c) $\frac{15}{16} - \frac{3}{4} =$

$$\frac{15}{16} - \frac{3 \cdot 4}{4 \cdot 4} = \frac{15}{16} - \frac{12}{16} = \boxed{\frac{3}{16}}$$

d) $\frac{18}{19} - \frac{2}{7} = \frac{18 \cdot 7}{19 \cdot 7} - \frac{2 \cdot 19}{7 \cdot 19}$

$$= \frac{126}{133} - \frac{38}{133} = \boxed{\frac{88}{133}}$$

Fraction Rules:

$$\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b} \quad \text{OR} \quad \frac{a}{b} - \frac{c}{b} = \frac{a-c}{b}$$

If the denominators are not the same, then you need to multiply the **individual pieces** by an expression to get the same denominator.

Example 1: Simplify the expression by adding or subtracting rational expressions with like denominators.

a. $\frac{7}{4x} + \frac{3}{4x}$

$$= \frac{7+3}{4x}$$

$$= \frac{10}{4x} = \boxed{\frac{5}{2x}}$$

b. $\frac{2}{x+3} - \frac{4}{x+3}$

$$= \boxed{\frac{-2}{x+3}}$$

c. $\frac{2x}{x+6} - \frac{5}{x+6}$

$$= \boxed{\frac{2x-5}{x+6}}$$

Example 2: Simplify the expression by adding or subtracting rational expressions with unlike denominators.

$$a. \frac{3}{4x^2} + \frac{2x}{12x}$$

$$\text{LCD: } 12x^2$$

$$= \frac{3}{4x^2} \cdot \frac{3}{3} + \frac{2x}{12x} \cdot \frac{x}{x}$$

$$= \frac{9}{12x^2} + \frac{2x^2}{12x^2}$$

$$= \boxed{\frac{9+2x^2}{12x^2}}$$

$$b. \frac{5}{6x^2} + \frac{x}{4x^2-12x}$$

$$\text{LCD: } 6x^2 \text{ and } 4x(x-3)$$

$$12x^2(x-3)$$

$$= \frac{5}{6x^2} \cdot \frac{2(x-3)}{2(x-3)} + \frac{x}{4x(x-3)} \cdot \frac{3x}{3x}$$

$$= \frac{10(x-3) + 3x^2}{12x^2(x-3)}$$

$$= \boxed{\frac{10x-30+3x^2}{12x^2(x-3)}}$$

$$c. \frac{4}{x^2} - \frac{8x-1}{2x^3}$$

$$\text{LCD: } 2x^3$$

$$= \frac{4}{x^2} \cdot \frac{2x}{2x} - \frac{(8x-1)}{2x^3}$$

$$= \frac{8x - 8x + 1}{2x^3}$$

$$= \boxed{\frac{1}{2x^3}}$$

$$d. \frac{4}{x^3} + \frac{x}{6x^3+3x^2}$$

$$\text{LCD: } x^3 \text{ and } 3x^2(2x+1)$$

$$3x^3(2x+1)$$

$$= \frac{4}{x^3} \cdot \frac{3(2x+1)}{3(2x+1)} + \frac{x}{3x^2(2x+1)} \cdot \frac{x}{x}$$

$$= \frac{12(2x+1) + x^2}{3x^3(2x+1)}$$

$$= \boxed{\frac{24x+12+x^2}{3x^3(2x+1)}}$$

$$e. \frac{x+1}{x^2+4x+4} - \frac{2}{x^2-4}$$

$$\text{LCD: } (x+2)(x+2) \text{ and } (x+2)(x-2)$$

$$(x+2)(x+2)(x-2)$$

$$= \frac{(x+1)(x-2)}{(x+2)(x+2)(x-2)} - \frac{2(x+2)}{(x+2)(x-2)(x+2)}$$

$$= \frac{x^2 - x - 2 - 2x - 4}{(x+2)^2(x-2)}$$

$$= \boxed{\frac{x^2 - 3x - 6}{(x+2)^2(x-2)}}$$

$$f. \frac{x+1}{x^2+6x+9} - \frac{1}{x^2-9}$$

$$\text{LCD: } (x+3)(x+3) \text{ and } (x+3)(x-3)$$

$$= \frac{(x+1)(x-3)}{(x+3)(x+3)(x-3)} - \frac{1(x+3)}{(x+3)(x-3)(x+3)}$$

$$= \frac{x^2 - 2x - 3 - x - 3}{(x+3)(x+3)(x-3)}$$

$$= \boxed{\frac{x^2 - 3x - 6}{(x+3)^2(x-3)}}$$