Name: Completed Notes S2

Unit 7 – Graphing and Modeling Trigonometric Functions PC

7-5 Applications

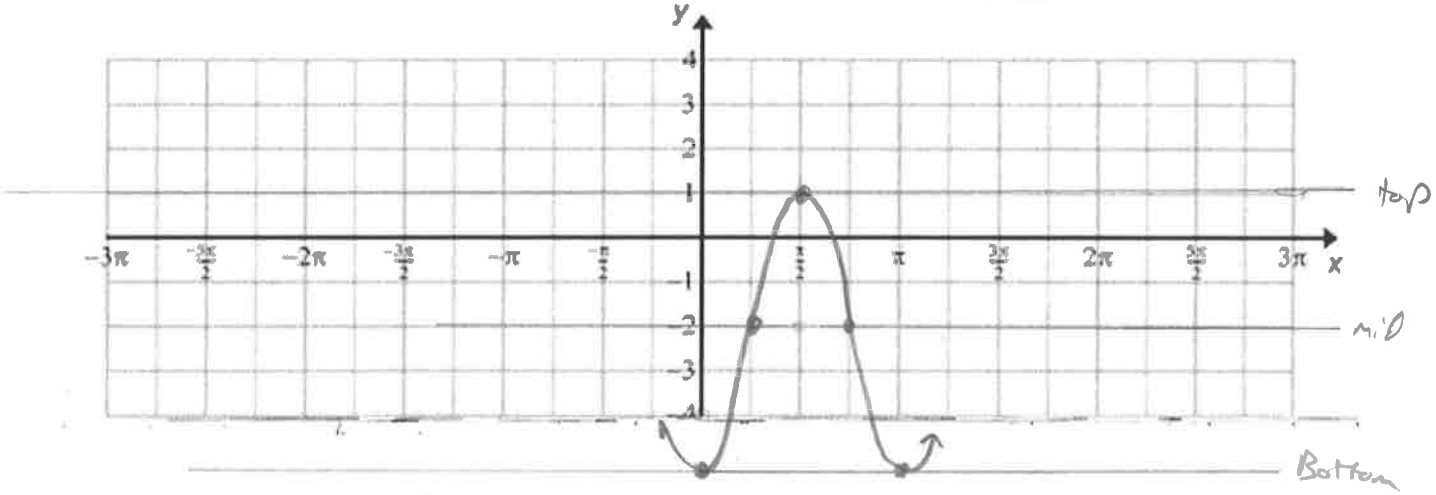
Objective(s):

1. Use trigonometric to model trigonometric functions to model real life situations.

Warm-up:

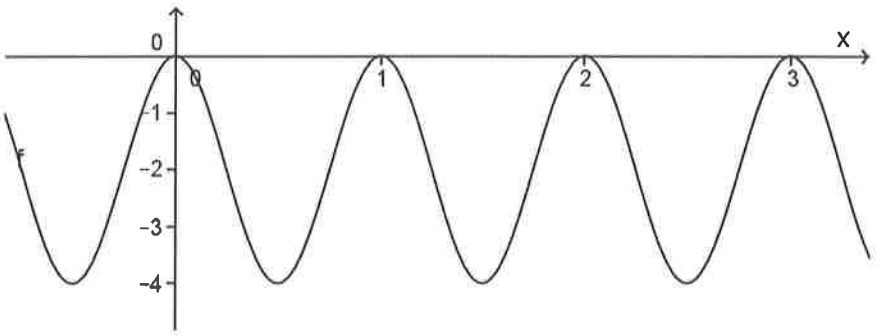
1. Graph the cosine equation: $y = -3\cos(2x) - 2$

Amplitude: 3 Period: π Midline: $y = -2$ Domain: $(-\infty, \infty)$ Range: $[-5, 1]$



2. Given the graph below, identify the period, midline and amplitude. Then write a possible equation that fits the graph both in terms of sine and cosine.

Amplitude: 2 Period: 1 b = $\frac{2\pi}{1} = 2\pi$ Midline: $y = -2$



In terms of Sine:

Phase shift: left $\frac{1}{4}$

Equation: $y = 2\sin(2\pi(x+1)) - 2$

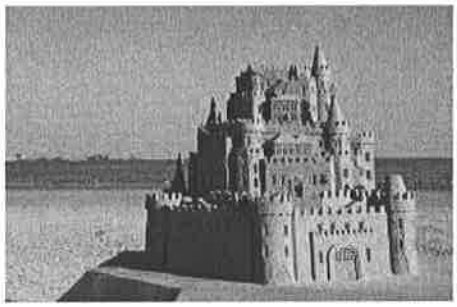
In terms of Cosine

Phase shift: none

Equation: $y = 2\cos(2\pi x) - 2$

APPLICATIONS

Perhaps you have built an elaborate sand castle at the beach only to have it get swept away by the incoming tide. Spring break is coming up and you are planning another trip to the beach. This time you decide to pay attention to the tides so that you can keep track of how much time you have to build and admire your sand castle.



You have a friend who is in calculus who will be going on spring break with you. You give your friend some data from the almanac about high tides along the ocean, as well as a contour map of the beach you intend to visit, and ask her to come up with an equation for the water level on the beach on the day of your trip. According to your friend's analysis, the water level on the beach will fit this equation:

$$f(t) = 20 \sin\left(\frac{\pi}{6}t\right)$$

In this equation, $f(t)$ represents how far the waterline is above or below its average position. The distance is measured in feet, and t represents the elapsed time (in hours) since midnight.

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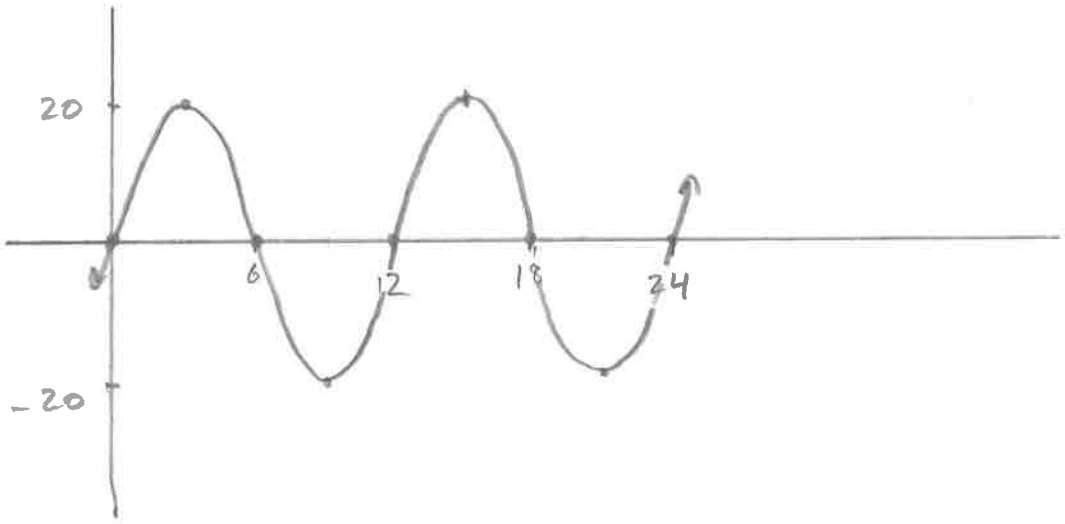
1. Graph the function below. Graph at least two full cycles.

amp = 20

per = $\frac{2\pi}{\frac{\pi}{6}}$

$= 2\pi \cdot \frac{6}{\pi}$

$= 12$



2. What is the highest up the beach (compared to its average position) that the waterline will be during the day? (This is called *high tide*.) What is the lowest that the waterline will be during the day? (This is called *low tide*.)

20 ft highest

-20 ft lowest

3. Suppose you plan to build your castle right on the average waterline just as the water has moved below that line. How much time will you have to build your castle before the incoming tide destroys your work?

From 6 am to 12 noon is 6 hours.

4. Suppose you want to build your castle 10 feet below the average waterline to take advantage of the damp sand. What is the maximum amount of time you will have to make your castle? How can you convince your friend that your answer is correct?

$-10 = 20\sin\left(\frac{\pi}{6}t\right)$

$-\frac{1}{2} = \sin\left(\frac{\pi}{6}t\right)$

$\sin^{-1}\left(-\frac{1}{2}\right) = \frac{\pi}{6}t$

$\frac{7\pi}{6} = \frac{\pi}{6}t$ or $\frac{11\pi}{6} = \frac{\pi}{6}t$

$t =$ 7 or 11 → this represents 7am and 11am,

so 4 hours.

5. Suppose you want to build your castle 15 feet above the average waterline to give you more time to admire your work. What is the maximum amount of time you will have to make your castle? How can you convince your friend that your answer is correct?

$\frac{15}{20} = \frac{20}{20}\sin\left(\frac{\pi}{6}t\right)$

$\frac{3}{4} = \sin\left(\frac{\pi}{6}t\right)$

$\sin^{-1}\left(\frac{3}{4}\right) = \frac{\pi}{6}t$

$t = \frac{6}{\pi}\sin^{-1}\left(\frac{3}{4}\right) \approx 1.6$ hours

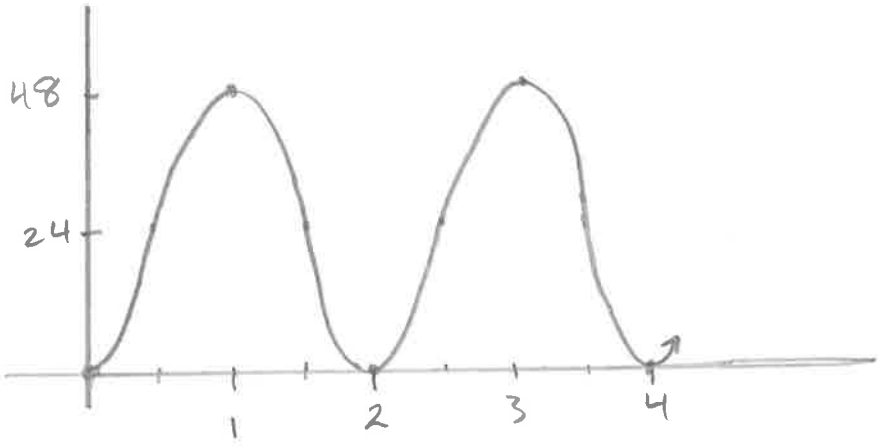
6. Suppose you decide you only need two hours to build and admire your castle. What is the lowest point on the beach where you can build it? How can you convince your friend that your answer is correct?

More applications:

Sam is riding his bike home from school one day and picks up a nail in his tire. The nail hits the ground every 2 seconds and reaches a maximum height of 48 cm (assume the tire does not deflate).

↳ period $b = \frac{2\pi}{2} = \pi$

1. Sketch a graph to model the height of the tire over 2 revolutions.



2. Write an equation to model the graph above. $y = -24\cos(\pi x) + 24$

C.) What is the height of the nail above the ground 2.5 seconds? 24 cm

$y = -24\cos(2.5\pi) + 24$

$y = -24\cos(\frac{5\pi}{2}) + 24$

$y = -24(0) + 24$

$y = 24$