

Directions: Show all work for full credit. DO NOT use a calculator for these problems, unless otherwise indicated.

For #1 – 5: Given below is a table of function values of  $h(x)$ . Approximate each of the following definite integrals using the indicated Riemann or Trapezoidal sum, using the indicated subintervals of equal length.

$x$	-3	-1	1	3	5	7	9
$h(x)$	5	2	-3	-7	-2	6	11

1.  $\int_{-3}^1 h(x) dx$  using two subintervals and a Left Hand Riemann sum.

$$\approx 2(5) + 2(2)$$

$$= \boxed{14}$$

2.  $\int_{-3}^9 h(x) dx$  using three subintervals and a Right Hand Riemann sum.

$$\approx 4(-3) + 4(-2) + 4(11)$$

$$= -12 - 8 + 44$$

$$= \boxed{24}$$

3.  $\int_{-3}^9 h(x) dx$  using three subintervals and a Midpoint Riemann sum

$$\approx 4(2) + 4(-7) + 4(6)$$

$$= 8 - 28 + 24$$

$$= \boxed{4}$$

4.  $\int_{-3}^3 h(x) dx$  using three subintervals and a Trapezoidal sum.

$$\approx \frac{1}{2}(2)(5+2) + \frac{1}{2}(2)(2+(-3)) + \frac{1}{2}(2)(-3+(-7))$$

$$= 7 + (-1) + (-10)$$

$$= \boxed{-4}$$

5.  $\int_{-3}^9 h(x) dx$  using six subintervals and a Trapezoidal sum.

$$\approx \frac{1}{2}(2) [5 + 2(2) + 2(-3) + 2(-7) + 2(-2) + 2(6) + 11]$$

$$= [5 + 4 - 6 - 14 - 4 + 12 + 11]$$

$$= \boxed{8}$$

For questions 6 and 7, approximate the definite integrals. Make a table of values showing your intervals that you used.

6. Approximate  $\int_0^{\pi} (2x \sin x) dx$  using four subintervals of equal length and a Right Hand Riemann sum.

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	$\pi$
f(x)	0	$\frac{\pi\sqrt{2}}{4}$	$\pi$	$\frac{3\pi\sqrt{2}}{4}$	0

$\frac{\pi}{2}, \frac{\sqrt{2}}{2}$        $\frac{3\pi}{2}, \frac{\sqrt{2}}{2}$

$$\approx \frac{\pi}{4} \left( \frac{\pi\sqrt{2}}{4} + \frac{4\pi}{4} + \frac{3\pi\sqrt{2}}{4} + 0 \right)$$

$$= \frac{\pi}{4} \left( \frac{4\pi + 4\pi\sqrt{2}}{4} \right)$$

$$= \frac{\pi}{4} (\pi + \pi\sqrt{2}) = \frac{\pi^2 + \pi^2\sqrt{2}}{4}$$

7. Approximate  $\int_{-2}^{10} (e^2 x^2) dx$  using four subintervals of equal length and a Trapezoidal sum.

x	-2	1	4	7	10
f(x)	$4e^2$	$e^2$	$16e^2$	$49e^2$	$100e^2$

$$\approx \frac{1}{2} (3) [4e^2 + 2(e^2) + 2(16e^2) + 2(49e^2) + 100e^2]$$

$$= \frac{3}{2} [236e^2]$$

$$= 354e^2$$

8. Given the table to the right, approximate

$$\int_{-2}^9 P(x) dx$$

using three subintervals and a Midpoint Riemann sum.

x	-2	0	1	3	5	8	9
P(x)	5	8	2	-4	-1	2	5

$$\approx 3(8) + 4(-4) + 4(2)$$

$$= 24 - 16 + 8$$

$$= 16$$

9. Given the table to the right, approximate

$$\int_{-2}^9 P(x) dx$$

using six subintervals and a Trapezoidal sum.

x	-2	0	1	3	5	8	9
P(x)	5	8	2	-4	-1	2	5

$$\approx \frac{1}{2} (2)(5+8) + \frac{1}{2} (1)(8+2) + \frac{1}{2} (2)(2-4) + \frac{1}{2} (2)(-4-1) + \frac{1}{2} (3)(-1+2) + \frac{1}{2} (1)(2+5)$$

$$= 13 + 5 - 2 - 5 + \frac{3}{2} + \frac{7}{2}$$

$$= 16$$